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PHYSICOMECHANICAL NATURE OF THE EFFECT OF MATERIAL STRENGTHENING FOR WHISKERS AND THIN FILAMENTS

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Numerous experiments by researchers at home and abroad have established the fact of a marked increase in the material strength of whiskers and thin filaments with a reduction in the characteristic size of them, i.e., the diameter or cross-sectional area. Existing hypotheses about the nature of this phenomenon are generally of a phenomenological character and they do not provide an explanation at the microlevel.

Concepts are advanced in this work which in the opinion of the authors make it possible to give a physicomechanical explanation of the strengthening effect.

1. The first researcher to establish reliably a connection between the material strength of a fine filament and its cross-sectional size was apparently Griffith [1]. Tests for breaking glass rods with diameters of 10-100 μm showed that the material strength of specimens achieved is significantly above the normal technical strength for a given material. Confirmation and further development of the results in [1] was obtained in tests on crystals of antimony, silicon, salt columns, and quartz filaments [2-5].

Subsequently experiments spread to metal whiskers and fine metal filaments. The results of tests by Herring and Holt with tin crystals are well-known. Experiments were carried out in [6-8] with copper and iron whiskers. For metals it was normally shown that with small test specimen diameters (about 1 μm) the material strength increases appraoching the theoretical strength. By theoretical strength σ_t we understand the limiting value of ultimate breaking resistance for material with an "ideal" structure.

A considerable amount of data has been accumulated recently in tests on crystals consisting of a structural base of various ceramics, and cemented and gypsum stones. These results are particularly reflected in [9-11].

By comparing and analyzing the numerous experimental results for different materials independent of the chemical nature and method of preparation it is possible to conclude that there is a clear connection between material strength and the cross-sectional size of a test whisker or a thin fiber.

2. Many researchers have attempted to explain the material strengthening effect described above for specimens of small size. In [1] an effect of a denser surface layer on the strength of thin filaments was suggested. However, subsequent tests [12] and particularly in [5] disproved this hypothesis. It was shown that even if some oriented layer on the surface of glass or quartz exists, its effect on the strength properties of specimens is hardly perceptible. Nonetheless, other concepts have been advanced in one form or another using the effect of a surface layer of a test specimen on material strength. For example, Ienckel-Münster [13] considered that forces of surface tension fulfill a specific strengthening role. He suggested an equation for describing the relationship between breaking, perimeter, and specimen cross section:

$$\sigma_f = a + bK/F.$$

Here σ_f is material ultimate breaking strength; K, F are specimen perimeter and cross-sectional area; a, b are constants determined from an experiment.

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Since in the case of a round cross section the ratio of perimeter to area is at a minimum, a conclusion follows from the Ienckel-Münster assumption: compared with any other specimen cylindrical specimens should be the strongest. However, this result was disproved subsequently by the tests of a number of researchers, and even by Ienckel-Münster himself testing specimens of an elliptical cross section. Thereby the idea of an effect on the material strength of thin filaments by surface tension was disproved.

Aleksandrov and Zhurkov attempted to connect the state of the surface layer with the strength of thin specimens [5]. They suggested the existence of defects of a different type, the most critical of which with greatest probability are encountered at the surface. As a calculation equation connecting the material strength and radius of a test filament the following relationship was suggested

$$\sigma_f = a + b/r + c,$$

where a is material technical strength realized in macrospecimens; b, c are constants determined by experiments.

The idea of Aleksandrov and Zhurkov was further developed in [14-16]. In [16] the connection of strength σ_f with a characteristic specimen size a was analyzed:

$$\sigma_f = \sigma_t (a_*/a)^i.$$

Here a_* is the size of a "defect-free" crystal with theoretical strength σ_t; i is index of the intensity of a reduction in strength connected in each specific case with experimental data given statistical treatment.

Concerning all of the hypotheses in question it is possible to say that they are essentially loose in nature. The reasons for strengthening treated phenomenologically from the position of statistical approaches are not revealed from a physicomechanical point of view.

3. In our opinion only the fact of the presence of defects in a "nonideal" specimen is indisputable in all modern concepts of the mechanism of material strengthening for whiskers and thin crystals. It appears to us that subsequent constructions connected with widely known versions confirming that the greater specimen volume and section, the greater the probability of the presence of "critical" crack-like defects in it and consequently the lower the material strength, are less convincing. In view of this it is of interest to consider this phenomenon from the position of the physicomechanical nature.

As is well-known from theory, crack-like defects lead to pronounced stress concentration. The latter increases infinitely at the mouth of a crack causing occurrence of the so-called feature of stress–strain state. Consequently, with any small but finite loads a specimen with a crack will develop zero strength, which is of course not actually observed. This contradiction is even noted by Griffith [1]. Proceeding from energy considerations he obtained an equation which connects the strength of a brittle materials σ_f with the size of a crack-like defect l:

$$\sigma_f = \sqrt{2\gamma E/\pi l} \qquad (3.1)$$

(E is normal elasticity modulus, γ is material specific surface energy).

Subsequently, after the publication of [17, 18], the prevailing position in crack mechanics and correspondingly removal of the contradiction noted above was the idea of a stress intensity factor. This specific characteristic of a stressed state (an especially unique factor) is connected with material strength by the equation

$$\sigma_f = K_c/\sqrt{\pi l}, \qquad (3.2)$$

where K_c is limiting value of intensity factor for a material established in an experiment. Normally the strength condition in this case is replaced by comparing the value found theoretically for K with K_c.

It can be seen that Eqs. (3.1) and (3.2) are similar in structure. They do not contain parameters which consider the scale effect on material strength. This scale factor may be introduced in a given case only by means of the reasoning mentioned above about the correlation between the specimen cross-sectional area or its volume and the probability of defects appearing. For this reason it seems to us that another method is desirable for constructing the connection between stress–strain state and material strength for thin filaments and crystals.

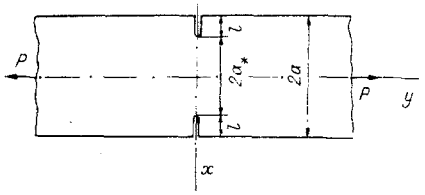

Fig. 1

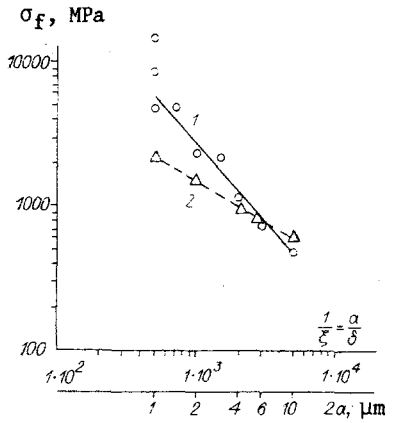

Fig. 2

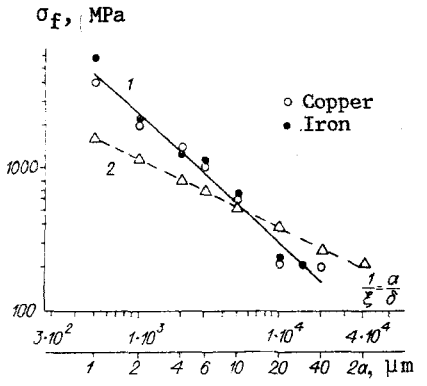

Fig. 3

4. Infinite stresses in the theoretical solution are a consequence of one of the main properties of the classical model of a solid deformed body, i.e., continuity. However, actual materials even at the atomic level are devoid of this property which in reality enters into the contradiction.

As shown in [19] this contradiction is avoided quite elegantly by changing over as required from stresses to forces which operate in the structural connections of a substance. We show that this approach is fruitful not only from the point of view of determining the critical length of a Griffith crack, but also in the scheme of explaining the reasons for an increase in the material strength of thin filaments.

We consider extension of a strip with thickness b and width 2a weakened by two through cracks each of length ℓ (Fig. 1). In essence this problem models some particular case of filamentary crystal material weakening by surface defects. The distribution of normal stresses in the connecting strip, determined by solving the plane problem of elasticity theory, is characterized by the equation

$$\sigma_y = P/\pi b \sqrt{a_*^2 - x^2}$$

(P is force extending the strip, $a_* = a - \ell$, b = 1). Subsequently, assuming that the crack has small dimensions ($\ell \ll a$) we assume that $a_* \cong a$. It can be seen that with $x \to a_*$ stress increases infinitely, i.e., there is a typical stepped property ahead of the crack tip.

Following Novozhilov we introduce the notion of a structural cell of material with characteristic size δ. We compute the elementary normal force ΔN arriving on a single structural connection isolated immediately ahead of the crack tip:

$$\Delta N = \int\int_{\Delta F} \sigma_y dF = \int_{a_*-\delta}^{a_*} \sigma_y(x)\, b dx = P/\pi \int_{a_*-\delta}^{a_*} dx/\sqrt{a_*^2 - x^2},$$
$$\Delta N = P/\pi\,[\pi/2 - \arcsin(1 - \delta/a_*)]. \qquad (4.1)$$

In those cases when $\delta \ll a_* \cong a$, i.e., the size of the structural cell is markedly less than the characteristic strip transverse dimension, the result (4.1) may be presented in simpler form

$$\Delta N \cong P/\pi \sqrt{2a_*} \int_{a_*-\delta}^{a_*} dx/\sqrt{a_* - x} \cong P\sqrt{2}\sqrt{\delta}/\pi\sqrt{a}. \qquad (4.2)$$

We introduce the notation for the relative dimension of a structural cell: ξ = δ/a. Then parameter ξ appears in Eqs. (4.1) and (4.2). A simpler force in structural connections may be calculated with uniform stress distribution: $\Delta N_{av} = (P/2ab)b\delta = P\xi/2$.

As a result of this it is possible to introduce the notation of elementary force concentration and to determine parameter α_N, i.e., the force concentration factor close to a particular point: $\alpha_N = \Delta N/\Delta N_{av}$. Depending on the particular type of the connection used [(4.1) or (4.2)] for α_N calculation equations are obtained:

$$\alpha_N = 2\,[\pi/2 - \arcsin(1 - \xi)]/\pi\xi$$

or

$$\alpha_N = 2\sqrt{2}/\pi\sqrt{\xi}. \qquad (4.3)$$

Since in real objects, for example filamentary crystals, ξ is small compared with unity, then in order to analyze experimental results we shall use expression (4.3). It is clear that with $\xi \to 0$, when the discrete model is converted into a continuous material, α_N increases infinitely and as a result it conforms with the normal stress concentration factor. An equation identical to (4.3) for α_N is also obtained for a filament of round cross section with an encircling notch-crack. In addition, it is necessary to note that the connection $\alpha_N(\xi)$ may be obtained not only for a crack, but also for any other form of stress concentration leading to the feature of a stress–strain state [20, 21].

Proceeding from the idea of a concentration factor for elementary forces it is possible to arrive at a qualitative an quantitative explanation of the effect of a reduction in filamentary crystal strength as their cracks grow. If $\alpha_N = 1$, i.e., the object degenerates into almost one or two defect-free structural connections, in a notional experiment we obtain $\sigma_f = \sigma_t$, i.e., ultimate breaking strength agrees with the theoretical strength.

As cross section increases and correspondingly α_N grows failure occurs with $\sigma_f < \sigma_t$, i.e., in a "critical" section where the maximum stresses distributed over the area of a structural cell reach σ_t. This relates to the condition $\alpha_N \sigma_f = \sigma_t$. Whence the main calculation equation develops for determining the technical material strength of a thin filament:

$$\sigma_f = \sigma_t / \alpha_N. \tag{4.4}$$

By using approximate relationship (4.3) in (4.4), we obtain

$$\sigma_f = \pi \sqrt{\xi} \sigma_t / 2\sqrt{2}. \tag{4.5}$$

For graphical analysis it is desirable to present Eq. (4.5) a little differently:

$$\sigma_f = \pi \sigma_t / 2\sqrt{2}\sqrt{1/\xi} = \pi \sigma_t / 2\sqrt{2}\sqrt{a/\delta}. \tag{4.6}$$

Since parameters σ_t and δ at the microlevel of the structure are constant for each specific material, then expression (4.6) on coordinate axes $\sigma_f \sim 1/\xi$ in fact determine the scale dependence of technical strength on the thickness of a filamentary crystal or thin filament.

Evaluation of theoretical results, found using Eq. (4.6), is carried out on the basis of comparison with known experimental data for the connection $\sigma_f = f(a)$ for filamentary crystals of Al_2O_3 [9], copper and iron [6-8], silicon [3], and glass filament [1]. Considering that these materials experience brittle failure, for the size of a structural cell at the microlevel it is possible to take some parameter of the atomic lattice. Thus for Al_2O_3, copper, iron, and silicon the size of an atomic bond δ = 100-200 nm, and for glass the size of a crystallite δ = 1500-2000 nm are used.

By using the relationship $\sigma_f = f(a)$ and corresponding values for δ we construct experimental curves 1 with $\sigma_f = f(1/\xi = a/\delta)$ for Al_2O_3 (Fig. 2), copper and iron (Fig. 3), silicon (Fig. 4), and glass filaments (Fig. 5).

Also by taking as theoretical strength σ_t the following values according to Kelly [22]: for Al_2O_3, σ_t = 47,000 MPa, for silicon σ_t = 32,000 MPa, for glass σ_t = 16,000 MPa; and according to Geminov [7] for copper and iron σ_t = 39,000 MPa, from Eq. (4.6) we construct analytical curves for the dependence 2 with $\sigma_f = f(1/\xi)$ for the materials in question (Figs. 2-5). Comparison of curves 1 and 2 shows that they are in good qualitative and quantitative agreement.

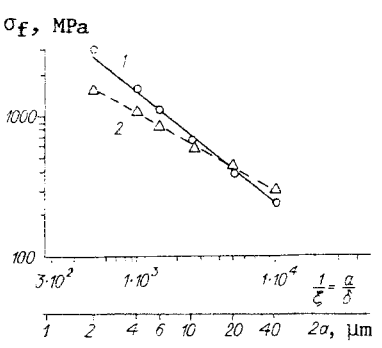

Fig. 4

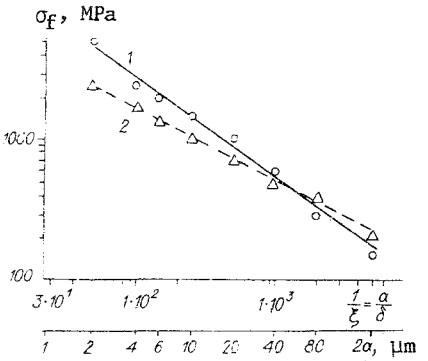

Fig. 5

In spite of the considerable number of publications connected with explaining the essence of the material strengthening effect with a change over to small size specimens the problem is still not finally resolved.

A hypothesis is formulated in the present work which makes it possible, in the opinion of the authors, to disclose the physicomechanical nature of the effect of an increase in material strength in whiskers and thin filaments. It appears that the approach in question may serve as a basis for a phenomenological hypothesis for the strength of brittle materials with a distinct crystalline structure in structural articles and those having a different type of stress–strain state.

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